

DL Reasoning

For CMCS498w

April 5, 2004

Inter-reductions

- Satisfiability to subsumption
 - Unsatisfiable(C) ==
 - C subClassOf Nothing
 - Satisfiable(C) ==
 - Unsatisfiable(\sim C)
 - not(Unsatisfiable(C))
 - Subsumption to unsatisfiability
 - C subClassOf D ==
 - Unsatisfiable(C & \sim D)
- With tableaux, you have a direct satisfiability test, but still use the Unsatisfiability of (C & \sim D). *Why?*
- Semi-decidability of one, leads to the (full) undecidability of the other. *Why?*

Tableau Algorithms

- Require full negation
- Basic Intuition
 - Manipulate the formula so than any (and all) contradictions are *obvious*
 - If your procedure is sure to reveal *all* contradictions and doesn't, you know there aren't any
- Basic Technique
 - Start with emptiable Tbox, empty Abox, and a concept C
 - Normalize(C)
 - Apply C to a random individual, x (e.g., x:C)
 - Apply the *expansion* rules to x:C until the tableau is complete

Normalizing C (for ALC)

- First, empty the Tbox
 - Tableau rules are not aware of “side constraints”
 - For non-general (“unfoldable”) TBoxes, just unfold (completely) C to C_u
- Second, put C_u into *negation normal form*
 - If atomic(A), $\text{nff}(A) == A$ and $\text{nff}(\sim A) == \sim A$
 - $\text{nff}(\sim(C \ \& \ D)) == \text{nff}(\sim C) \vee \text{nff}(\sim D)$
 - $\text{nff}(\sim(C \ \vee \ D)) == \text{nff}(\sim C) \ \& \ \text{nff}(\sim D)$
 - $\text{nff}(\sim \forall P.C) == \exists P.\text{nff}(\sim C)$
 - $\text{nff}(\sim \exists P.C) == \forall P.\text{nff}(\sim C)$

Tableau Rules

- One rule for each constructor
 - $\&$ -rule
 - \vee -rule
 - \forall -rule
 - \exists -rule
- \vee -rule introduces (lots) of non-determinism (or-branches)
- \exists -rule introduces *successors* (and-branches)
- A *clash* is where some branch has both C and $\sim C$

Next time

- Reading
 - An Overview of Tableau Algorithms for DLs
 - <http://www.inf.unibz.it/~franconi/dl/course/articles/baader-tableaux.ps.gz>
 - Sections 1, 2, 3, 5 for review
 - Section 4.1 (you can skim 4.2)
 - Sections 6.2.1, 6.2.2
 - Should finish From SHIQ and RDFS to OWL
- ALCN (i.e., number restrictions)
- General TBoxes (hence blocking)
- Role Hierarchy with Transitive roles (SH, mostly)
- Datatypes (ALC(D)) (No reading for that)